LADDER NETWORK DESIGN THROUGH OPTIMIZATION [1]

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[1] The paper is dedicated to the memory of prof. T. Cholewicki

Abstract

The paper presents the algorithm for optimal approximation of the given frequency characteristics by the function satisfying the realizability conditions of LC resistively terminated ladder network. The developed program enables the analysis and design of the ladder, delivering as a result the optimal values of the circuit parameters and the transfer function of the circuit. It may be also applied to the reduction of the ladder at the minimum error of the frequency characteristics. The solution uses the Gauss-Newton optimization algorithm in cooperation with the so called continuant method for the analysis of the ladder. The numerical results of the design of some chosen filters are also presented and discussed.

1 INTRODUCTION

The usual form of construction of higher order filters is to simulate a passive LC doubly resistively terminated prototype. The simulation is done by using active RC networks containing either operational amplifiers or other active devices like controlled sources. The active network obtained in this way retains good sensitivity properties of the prototype from which the most important is the low sensitivity in the passband. Thanks to this attractive feature of the ladder it is usually the prototype for active RC, digital or more recently switched capacitor filter fabrications [1 - 3, 5 - 9, 16 17].

The aim of the paper is to present the computer aided approach to the design of the ladder network in the frequency domain to satisfy the preliminary specifications. Although the problem in not new and was treated in many papers [18, 11] the applications of the newest achievements in mathematical programming and of special mathematical description of the ladder, based on the theory of so called continuants [2, 3, 4, 16, 5, 20, 13, 12] opens some new possibilities in both, analysis and design of that kind of networks.

The paper will present the algorithm that enables the optimal approximation of the given input-output data (frequency characteristics) that satisfy the realizability conditions of LC resistively terminated ladder network and at the same time gives this network elements values. The program allows the optimal design of the circuit by minimizing the error between the desired frequency characteristic and the actual one. It may be also applied to the reduction of the ladder at the minimum error of the frequency characteristics. Using only the part of the program associated with the analysis, we can also investigate directly the properties of the ladder network, especially the sensitivity features. The numerical results of some examplary characteristics will be also presented and discussed.

2 STATEMENT OF THE PROBLEM

Assume that the input-output relationship to be simulated is given in the form of frequency characteristics: magnitude, phase or both. Our aim is to find the rational function corresponding to the passive ladder network that simulates the input-output data in the optimal way, where the optimality is defined in the least squares sense. The functions beeing considered here may belong either to the voltage transfer or input immittance descriptions of the network. Let $\boldsymbol{\Omega}$ denotes the set of frequences $\boldsymbol{\Omega} = [\omega_1, \ \omega_2, \ \cdots, \ \omega_n]^T$ over which the frequency characteristics should be adjusted to fit the reference (required) one. Let $\mathbf{y}$ denotes the reference vector of the frequency characteristic $\mathbf{y} = [y_1, \ y_2, \cdots, \ y_n]^T$, magnitude or phase, corresponding to the given frequency set $\boldsymbol{\Omega}$ and $\mathbf{x}$ is the vector corresponding to the parameters of the circuit, (elements of the ladder network) $\mathbf{x} = [x_1, \ x_2, \ \cdots, \ x_m]^T$. The number n of samples of frequency characteristic is usually much bigger than the number m of adjusted design parameters. The objective function $\xi(\mathbf{x}, \boldsymbol{\Omega})$ is the sum of the squared errors over the frequency range given by the vector $\boldsymbol{\Omega}$, where the error is understood as the difference between the required response defined in the vector $\mathbf{y}$ and the actual one denoted here by $\mathbf{f}(\mathbf{x}, \boldsymbol{\Omega})$. If we denote by $\mathbf{r}$ the residuum vector, $\mathbf{r} = [r_1, \ r_2, \ \cdots, r_n]^T$, where

$$\mathbf{r} = \mathbf{f}(\mathbf{x}, \boldsymbol{\Omega}) - \mathbf{y} \tag{1}$$

then the objective (cost) function subject to the minimization (denoted here for simplicity as $\xi(\mathbf{x})$) is defined by

$$\xi(\mathbf{x}) = 0.5\mathbf{r}^T\mathbf{W}\mathbf{r} = 0.5\sum_{i=1}^{n} w_i r_i^2 \tag{2}$$

where $\mathbf{W}$ is the diagonal weight matrix adjusted by the designer to emphesize the chosen frequency range, with the boundary constraints imposed on the optimized variables $\mathbf{x}$, where these constraints are of the form $\mathbf{x}_{min} \leq \mathbf{x} \leq \mathbf{x}_{max}$.

A special attention should be paid to the case when both magnitude and phase characteristics are fitted. In such case there is usually different scale for amplitude and different one for phase. If we denote by the subscript m the magnitude and by p the phase, the required, actual and residual vectors of magnitude and phase characteristics assume the notations $\mathbf{y}_m$, $\mathbf{f}_m(\mathbf{x}, \boldsymbol{\Omega})$, $\mathbf{r}_m = (\mathbf{f}_m(\mathbf{x}, \boldsymbol{\Omega}) - \mathbf{y}_m)$ and $\mathbf{y}_p$, $\mathbf{f}_p(\mathbf{x}, \boldsymbol{\Omega})$, $\mathbf{r}_p = (\mathbf{f}_p(\mathbf{x}, \boldsymbol{\Omega}) - \mathbf{y}_p)$, respectively. If we take simultaneously into account the magnitude and phase characteristics we have to adjust the weight matrices $\mathbf{W}_m$ and $\mathbf{W}_p$ emphesizing not only the chosen range of frequencies but also indicating which of them, magnitude or phase, should be more important. The objective function under minimization is then defined as

$$\xi(\mathbf{x}) = 0.5\mathbf{r}_m^T\mathbf{W}_m\mathbf{r_m} + 0.5\mathbf{r}_p^T\mathbf{W}_p\mathbf{r_p} \tag{3}$$

As a result, irrespective of the specified characteristic, the problem solution is transformed to the nonlinear optimization problem with some boundary constraints imposed on the design variables. The solution of it requires application of the proper optimization method cooperating with the efficient analysis of the ladder. Usually only gradient methods are robust enough to deal with the problem.

3 OPTIMIZATION PROCEDURE

The optimization problem defined in the previous section was solved by authors using the gradient optimization algorithm due to Gauss-Newton with the Levenberg-Marquardt regularization [14, 8]. In the minimization process we use the information not only about the cost function $\xi(\mathbf{x})$ but also about its gradient $\nabla\xi(\mathbf{x}) = \partial(\xi(\mathbf{x})/\partial\mathbf{x}$ and hessian $\nabla^2\xi(\mathbf{x}) = \partial^2(\xi(\mathbf{x})/\partial\mathbf{x}^2$. Denoting the jacobian matrix corresponding to $\mathbf{r}(\mathbf{x})$ by $\mathbf{J}(\mathbf{x})$, where $[\mathbf{J}(\mathbf{x})]_{i,j} = \partial r_i/\partial x_j$ the gradient vector $\nabla\xi(\mathbf{x})$ is given by

$$\nabla\xi(\mathbf{x}) = \frac{\partial}{\partial\mathbf{x}}\left[0.5\sum_{i=1}^{n} w_i r_i^2\right] = \sum_{i=1}^{n} w_i r_i \nabla r_i^T \tag{4}$$

and in the matrix notation

$$\nabla\xi(\mathbf{x}) = \mathbf{J}^T(\mathbf{x})\mathbf{W}\mathbf{r}(\mathbf{x}) \tag{5}$$

Similarly after differentiation of the gradient we get the hessian matrix which may be written in the form

$$\nabla^2\xi(\mathbf{x}) = \sum_{i=1}^{n}\left[w_i\nabla r_i\nabla r_i^T + w_i r_i\nabla^2 r_i^T\right] \tag{6}$$

or in the short matrix notation

$$\nabla^2\xi = \mathbf{J}^T(\mathbf{x})\mathbf{W}\mathbf{J}(\mathbf{x}) + \mathbf{S}(\mathbf{x}) \tag{7}$$

where $\mathbf{S}(\mathbf{x})$ is the matrix of the second derivatives of the residuum. The determination of the matrix $\mathbf{S}(\mathbf{x})$ is a very complex and time consuming task. Instead of direct calculation of it different kinds of approximations are applied, from which the most effective is the Gauss - Newton with Levenberg - Marquardt regularization. In this approach the hessian matrix is approximated using only information about the first derivative of the residuum matrix. The matrix $\mathbf{S}(\mathbf{x})$ is substituted by the diagonal matrix $\mathbf{M}$ of the elements proportional to the so called Levenberg-Marquardt parameter v, $\mathbf{M} = v\mathbf{1}$, where $\mathbf{1}$ is the identity matrix. Thus

$$\nabla^2 \xi \cong \mathbf{J}^T(\mathbf{x})\mathbf{W}\mathbf{J}(\mathbf{x}) + v\mathbf{1} \tag{8}$$

The subsequent steps in the optimization routine are formed according to the Newton optimization formula

$$\mathbf{x}_{k+1} = \mathbf{x}_k - \left[\nabla^2 \xi(\mathbf{x}_k)\right]^{-1} \nabla \xi(\mathbf{x}_k) \tag{9}$$

where $\mathbf{x}_k$ is the preceeding, known value of the optimized vector $\mathbf{x}$. Note that $v = 0$ reduces (8) to the classical Gauss-Newton scheme with quadratic convergence near optimum but poor behaviour if actual parameters forming vector $\mathbf{x}$ are far from optimal. If v is much larger than the eigenvalues of the matrix $\mathbf{J}^T\mathbf{W}\mathbf{J}$ then (8) describes small steepest descent step of the length $\| d\mathbf{x} \| = \| \nabla(\mathbf{x}) \| / v$. In general, steepest descent converges for poor starting values of $\mathbf{x}$ but requires rather lengthy solution time. Thus in the proposed approach the initial value of the regularization parameter v is large and will decrease to zero as the optimum is approached. The choice of v is very critical and depends on the actual value of the objective function $\xi_j = \xi(\mathbf{x}_j)$ and the previous one, $\xi_{j-1} = \xi(\mathbf{x}_{j-1})$. The strategy of changing v is as follows [14]. Let γ be some positive factor of $\gamma > 1$ (in practical implementation on PC we have assumed $\gamma = 10$). Then

- if $\xi(v_{j-1}/\gamma) \leq \xi_j$ then $v_j = v_{j-1}/\gamma$
- if $\xi(v_{j-1}/\gamma) > \xi_j$ AND $\xi(v_{j-1}) \leq \xi_j$ then $v_j = v_{j-1}$
- if $\xi(v_{j-1}/\gamma) > \xi_j$ AND $\xi(v_{j-1}) > \xi_j$ then increase the value of v by repeating w times the succesive multiplications of factor γ until $\xi(v_{j-1}\gamma^w) \leq \xi_j$ and let then

$v_j = v_{j-1}\gamma^w$. This procedure of changing the value of v is carried out until the actual value of the parameter q defined in the way

$$q = \frac{\xi_j - \xi_{j-1}}{\boldsymbol{\Delta} x^T \nabla \xi(\mathbf{x}) + 0.5 \boldsymbol{\Delta} x^T \nabla^2 \xi(\mathbf{x}) \boldsymbol{\Delta}\mathbf{x}} \tag{10}$$

reaches the level of unity. At the value of q equal one the real change of the objective function at the last optimization step reaches the possible theoretical one and further change of v has no sense.

The optimization steps should gradually reduce the value of v to zero. If its actual value is negligible by comparisonn with 1.0 to the number of significant figures (in practical implementation the minimal value of v was equal $10E - 8$) put $v = 0$ and start pure Gauss-Newton iteration scheme until sufficient accuracy is achieved. The practical implementation of the algorithm on IBM PC was proven to be quite reliable and robust in application to the design of the ladder networks. The satisfactory solution to the optimization problems have been found for relative wide ranges of starting values of the design variables forming vector $\mathbf{x}$.

4 THE ANALYSIS ALGORITHM OF THE LADDER BASED ON CONTINUANTS

The efficient implementation of the optimization algorithm requires application of quick and efficient analysis method, because the analysis procedure, beeing the integral part of the package, is called up many times during the optimization routine. The applied solution to the analysis problem uses the so called continuants [4, 16, 5, 20, 12] to express all variables of the circuit in an explicit way. Let us assume that the circuit under consideration is the ladder network terminated on both sides by the resistances as it is shown in Fig.1. According to [3, 5, 20, 12] the voltage transfer and input impedance functions are expressed in the following way

$$T_v = \frac{V_{out}}{V_{in}} = \frac{1}{A_{11} + R_s A_{21} + G_o A_{12} + R_s G_o A_{22}} \tag{11}$$

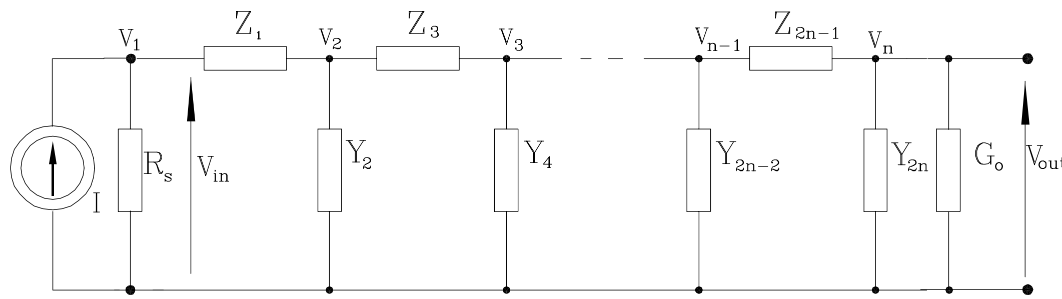

Figure 1: *The general form of ladder network with resistive termination*

$$Z_{in} = \frac{V_{in}}{I_{in}} = \frac{A_{11} + R_s A_{21} + G_o A_{12} + R_s G_o A_{22}}{A_{21} + G_o A_{22}} \tag{12}$$

where R_s and G_o denote the resistive terminations of the ladder while A_{ij} $(i, j = 1,\ 2)$ are the subparameters of the chain matrix **A** that are expressed using so called continuants [3, 16, 5, 12] defined in the following way

$$\begin{aligned} A_{11} &= \mathcal{K}(1, 2n) = (a_1..a_{2n}) \\ A_{12} &= \mathcal{K}(1, 2n-1) = (a_1..a_{2n-1}) \\ A_{21} &= \mathcal{K}(2, 2n) = (a_2..a_{2n}) \\ A_{22} &= \mathcal{K}(2, 2n-1) = (a_2..a_{2n-1}) \end{aligned} \tag{13}$$

The terms in brackets $(a_1..a_{2n})$ represent the immittances of the ladder network of Fig. 1, for example $a_1 = Z_1$, $a_2 = Y_2$, $a_3 = Z_3$, etc. They may be given either numerically or as a fuction of any variable, for instance complex frequency s. The continuants appearing in (13) can be derived in an algorithmic way using the following recurrence relation [16, 5, 20, 12].

$$\mathcal{K}(i, j) = a_j \mathcal{K}(i, j-1) + \mathcal{K}(i, j-2) \tag{14}$$

for $j = i, i+1, i+2....$ and assuming that $\mathcal{K}(i, i-1) = 1$ and $\mathcal{K}(i, i-2) = 0$. In the case of expressions (11),(12) and (13) we have $i = 1$ or $i = 2$. For example $\mathcal{K}(1,1) = (a_1 \,..\, a_1) = a_1$, $\mathcal{K}(1,2) = (a_1 \,..\, a_2) = a_1 a_2 + 1$, $\mathcal{K}(1,3) = (a_1 \,..\, a_3) =$

$a_1a_2a_3 + a_1 + a_3$, etc. Assuming that the network of Fig. 1 is the lossless LC we can easily obtain the rational functions of complex frequency variable s corresponding to the voltage transfer or input impedance functions. Moreover applying the properties of continuants [5, 20, 12] we can derive the explicit form of the sensitivity functions needed in our optimization algorithm. Taking into account the basic property of the continuants [16, 5]

$$\frac{d\mathcal{K}(i,j)}{da_k} = \mathcal{K}(i,k-1)\mathcal{K}(i,k+1) \tag{15}$$

for $i \leq k \leq j$ and zero elsewhere, we can derive the arbitrary derivative of any transfer function. For example in the case of voltage transfer function T_v described by (11) we get

$$\frac{dT_v}{da_k} = -T_v^2\left[\frac{d\mathcal{K}(1,2n)}{da_k} + R_s\frac{d\mathcal{K}(2,2n)}{da_k} + G_o\frac{d\mathcal{K}(1,2n-1)}{da_k} + R_sG_o\frac{d\mathcal{K}(2,2n-1)}{da_k}\right] \tag{16}$$

and finally after substitution of the corresponding relations we get

$$\begin{aligned}\frac{dT_v}{da_k} &= -T_v^2\left[\mathcal{K}(1,k-1)\mathcal{K}(k+1,2n) + R_s\mathcal{K}(2,k-1)\mathcal{K}(k+1,2n) + \right.\\ &+ \left. G_o\mathcal{K}(1,k-1)\mathcal{K}(k+1,2n-1) + R_sG_o\mathcal{K}(2,k-1)\mathcal{K}(k+1,2n-1)\right]\end{aligned} \tag{17}$$

As it is seen all solutions including transfer functions and sensitivities are expressed in an uniform, explicit way using the formulation of continuants. For each type of ladder these continuants are generated only once and then called up as many times as it is needed by the optimization program. This is the main reason of efficiency of the proposed method.

From the practical point of view the most important is the lowpass filter because it is usually the prototype for the transformations to the other types of the filters. At this stage two most representative classes of lowpass ladder networks we recognize:

1. Single inductors in the series branches and single capacitors in the shunt branches, for which

   $$a_{2i-1} = sL_{2n-1}, \quad a_{2i} = sC_{2i}$$

   ($i = 1,\ 2,\ ...,\ n$). These components contribute to arbitrary placement of poles but infinite zeros only; the finite zeros can not be created in this way.

2. Parallell LC circuits in series branches and simple capacitors in the shunt branches, for which we have

$$a_{2i-1} = \frac{\frac{1}{C_{2i-1}}s}{s^2 + \frac{1}{L_{2i-1}C_{2i-1}}}, \quad a_{2i} = sC_{2i}$$

This selection of components contribute not only to the arbitrary placement of the poles but also to the finite zeros placed on the $j\omega$ axis.

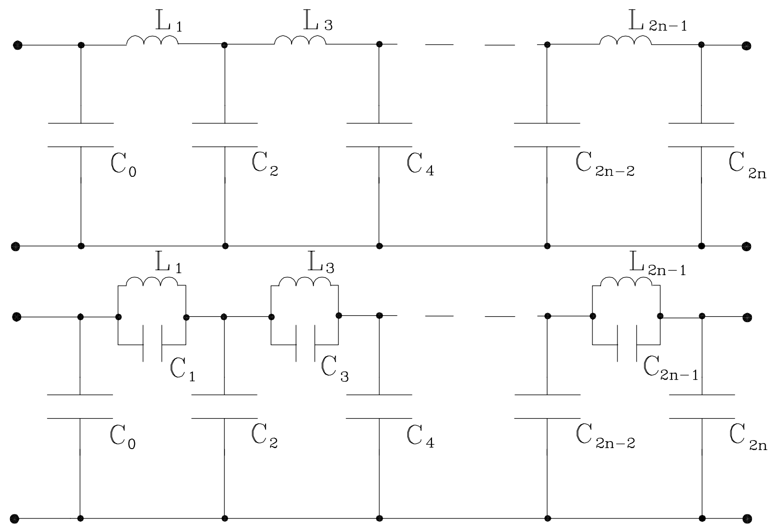

Figure 2: *The ladder LC network of a) L,C branches, b) LC, C branches*

Fig. 2a and 2b present both discussed types of the ladder network. Fig. 2a corresponds to the infinite and Fig.2b to the finite zeros of the transfer function. For the other classes of ladder, not discussed here, all network functions are calculated in the same way by assuming proper form of the coefficients a_i .

5 EXAMPLES OF DESIGN

The described above procedures of optimization and algorithmic analysis of ladder network were combined together and implemented on PC microcomputer. We will present here two most important types of the lossless ladder networks: one containing series L and shunt C (Fig.2a) and the second of shunt C and parallel LC resonator as the series elements (Fig.2b) . Program calculates all continuants needed in determination of transfer and input impedance functions as well as all sensitivities beeing used in the optimization process. It adjusts the ladder network elements to fit the required frequency characteristics: the magnitude one, the phase one or both together. The input file should contain the data, describing the required coordinates of the adjusted characteristic. These data may be associated with the voltage transfer or input immittance of the ladder. The computer delivers the optimal values of all inductances and capacitances of the network and also the symbolic form of the solution as the rational function of the complex frequency s. The program enables also the reduction of the system, i.e., the approximation of the higher order filters by the lower order model, realizable in the double resistively terminated LC ladder. The reduction preserves the best fitting of the frequency characteristic in the least squares sense. Below two examples illustrating different kinds of operations of the program will be presented. The first one is to design the lowpass ladder network satisfying the following specifications of the magnitude characteristic: the passband attenuation of $6dB$ with the ripple of $0.15dB$, the passband edge of $0.16rad$, the minimum attenuation at stopband of $40dB$ and the transit frequency range from passband to stopband not bigger than $0.02rad$. To satisfy the technical specifications the $7th$ degree filter of the structure presented in Fig.2b has been applied. As the design variables we have chosen the shunt capacitances C_0, C_2, C_4, C_6, the resonant frequencies (attenuation poles) of the series arms $1/L_1C_1$, $1/L_3C_3$, $1/L_5C_5$ and also $1/C_1$, $1/C_3$, $1/C_5$. The terminating resistances R_s and G_o were taken as unity. The most influential and sensitive are the starting values of the attenuation poles. To provide the convergence of the optimization process to the correct solution it was enough to place them in the required attenuation range of the given magnitude characteristic and this is very simple task for the designer. The guesses of starting values for the other design variables (capacitances) were not important for the optimization process of the ladder (assuming that their numerical values were not senseless); they caused only difference in the number of iterations.

Fig. 3a presents the overall and Fig. 3b the detailed passband magnitude response

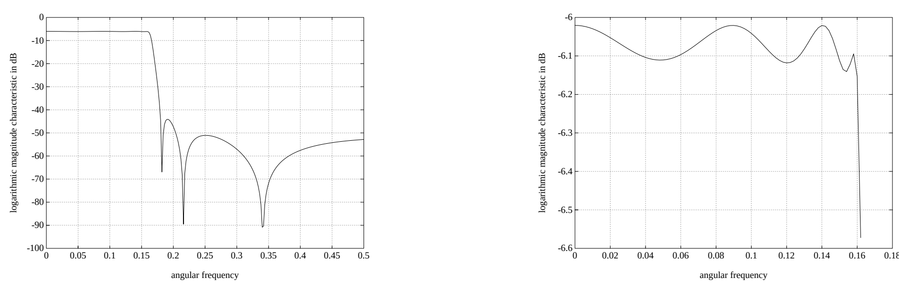

Figure 3: *The amplitude response of the filter a) the logarithmic characteristic in the passband and in the stopband, b) the detailed passband range*

of a $7th$ order ladder filter, designed by the program. As it is seen from the figure the response satisfies the required specifications. It shold be noted that the least squares error, counted for 25 frequency points evenly distributed within the frequency range, has been reduced from the starting value of 0.12169 to the final one (the minimum) of $0.18541E-3$ after only few iterations and the exact number of iterations was dependent on the starting values of the design parameters. At the starting values of $C_0 = 1.0$, $\frac{1}{C_1} = 1.0$, $\frac{1}{L_1 C_1} = 1.5$, $C_2 = 1.0$, $\frac{1}{C_3} = 1.0$, $\frac{1}{L_3 C_3} = 2.0$, $C_4 = 1.0$, $\frac{1}{C_5} = 1.0$, $\frac{1}{L_5 C_5} = 2.5$ and $C_6 = 1.0$ the optimal results, that correspond to $C_0 = 0.4029$, $C_1 = 1.5791$, $L_1 = 0.4831$, $C_2 = 1.2203$, $C_3 = 0.5365$, $L_3 = 1.0135$, $C_4 = 1.5534$, $C_5 = 0.1809$, $L_5 = 1.2044$, $C_6 = 0.9827$ were obtained after 9 iterations and the whole process has used only few second on microcomputer to get the optimal values of the filter parameters.

As the second example consider the design of the lowpass ladder network of Fig.2a satisfying simultaneously the specifications imposed on the magnitude and phase characteristics in the passband extending from $\omega = 0$ to $\omega = 1$. The filter is supposed to be of the linear phase in the passband of the group delay $\theta = -d\{argT(w)\}/dw = 3.15$ and the attenuation in the passband not greater than $8dB$. This time the circuit structure presented in Fig. 2a of $8th$ order has been chosen. As the design variables we put the inductances and capacitances i.e., $\mathbf{x}^T = [L_1,\ C_2,\ L_3,\ C_4,\ L_5,\ C_6,\ L_7,\ C_8]$ and the resistive terminations have been kept constant and equal to unity. The process of optimization has resulted in the optimal values of the filter parameters. The least mean square error was reduced from the starting value of 69.6151 to the minimum one of $0.7358E-5$ after 19 iterations. The starting values of the design parameters were assumed as follows: $L_1 = 1.0$, $C_2 = 2.0$, $L_3 = 3.0$, $C_4 = 4.0$, $L_5 = 5.0$, $C_6 = 6.0$, $L_7 = 7.0$, $C_8 = 8.0$ and the optimal ones were found equal: $L_1 = 1.2429$, $C_2 = 0.6849$, $L_3 = 1.7483$, $C_4 = 0.00105$, $L_5 = 0.02828$, $C_6 = 1.3896$, $L_7 = 0.8960$, $C_8 = 03015$. The transfer function $T(s) = N(s)/D(s)$ corresponding to these parameters is equal to

$$N(s) = 1.0$$

$$D(s) = 2.0{+}6.29s{+}9.31s^2{+}8.66s^3{+}5.44s^4{+}2.34s^5{+}0.568s^6{+}0.686E{-}4s^7{+}0.167E{-}4s^8$$

The parameters of the filter at the start of optimization procedure were adjusted arbitrarily to demonstrate the robustness of the program . Hence the differences between

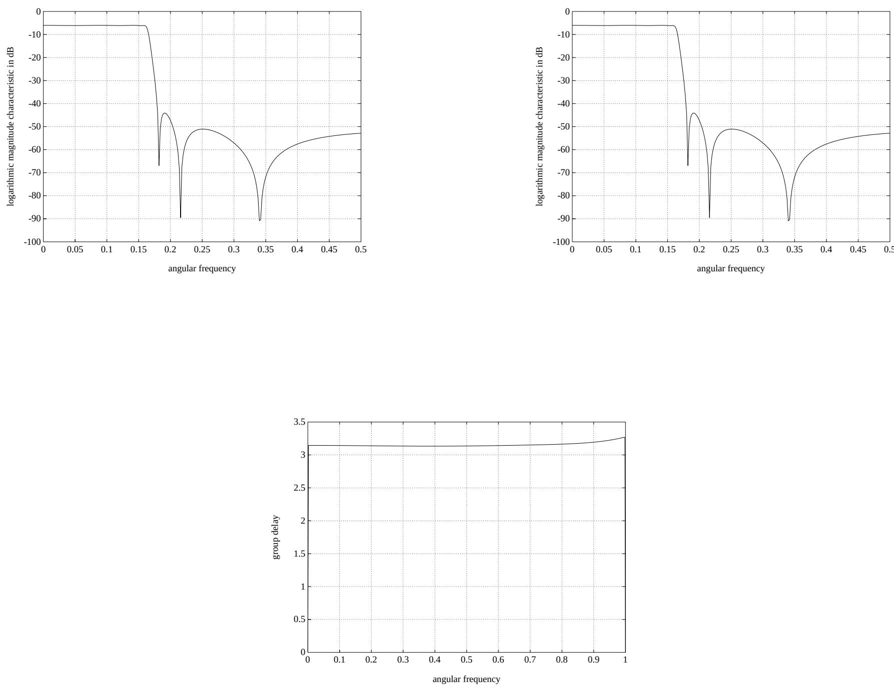

Figure 4: *The frequency characteristics of the optimal filter in the passband: a) the logarithmic magnitude response, b) the phase response, c) the delay response*

respective required and actual frequency characteristics at the start were large and the initial mean square error was big.

Fig.4 a, b and c present the obtained frequency charcteristics of the optimal filter satifying the requirements imposed on the filter. Fig. 4a presents the magnitude, Fig. 4b - the phase and Fig. 4c - the group delay characteristics of the obtained filter. The group delay is departed from the required value of less than 4.3% in the whole passband range, while the attenuation of the magnitude characteristic is kept below $7.8dB$. Thus the accuracy of the design program has been fully achieved.

6 CONCLUSIONS

The paper has described the optimization approach to the design of the filters realized in the ladder network structure. The Gauss-Newton method with Levenberg-Marquardt regularization was applied. The cooperation of this algorithm with the efficient analysis method, based on application of continuants, results in a quick and robust optimization package. The presented program enables the approximation of the given input-output data (frequency characteristics) in such a way that the resulting circuit is realizable in the resistively terminated LC ladder network and the parameters of this ladder are automatically calculated. The program may be also used to the reduction of the ladder type systems, that is, the reduced order approximation, while keeping the overall error at the assumed, acceptable level. The numerical results presented in the paper confirm the usefulness of the solution and its applicability to the design of the ladder type systems.

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